UNDERGRADUATE MATHEMATICS CLUBS.

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CLUB ACTIVITIES.

DENISON MATHEMATICS CLUB, Denison University, Granville, Ohio.

This Club came into existence through well attended meetings of an informal nature in 1915 shortly after Professor Forbes B. Wiley went to Denison University as head of the department of mathematics. In January, 1916, a constitution was approved and a club formally organized "to bring before its members matters of interest in mathematics that are not regularly discussed in courses offered in the curriculum." Any college student or member of the faculty who desires to join the club is eligible to do so. The average attendance at meetings last year was about 25 as against 30 the year before.

Officers 1917–18: President, Grace Jefferson '18; vice-president, Ruth Phillips '19; secretary-treasurer, Sterling Abell '20; assistant secretary-treasurer, Lawrence Curl '20. The program committee consists of these officers together with the head of the department.

October 3, 1916: Election of officers for the year.

November 7: The purposes of the club explained by the president, and various short topics presented to illustrate to new-comers some of the possibilities of such an organization.

November 21: "Problems in the mathematics of astronomy" by Professor Wiley.

December 19: "History of the Denison Mathematics Club"[1] by Marie Tilbe '17; "Integral curves" by Professor Anna B. Peckham.

January 23, 1917: "Mathematical societies and journals" by Professor Wiley; attention was drawn to problems in this MONTHLY.

February 20: Solution of problems proposed at the previous meeting by Lawrence Curl '20 and Virgil Traxler '19; "Spherical projection" by Harlan C. Reynolds '17.

March 6: "The possible use of parallel axes in the plane of intersecting axes of coördinates" by Professor Wiley.

March 20: "The concept infinity" by Professor Paul Biefeld of the astronomy department.

April 17: "Hyperbolic functions" by Professor Wiley.

May 1: Officers for 1917–18 elected.

September 25: "The probability function" by Professor Wiley.

October 23: "The fourth dimension" discussed by Sterling Abell '20, George Read '18, Clifford Marshall '18 and Charles T. Bumer '19.

November 6: "Theorems of elementary number theory" by Professor Wiley.

November 20: "Inversion" by Professor Peckham.

[1] Referring, presumably, to meetings before the formal organization.

December 18: "Constructions with compasses alone and with ruler alone" by George T. Street Jr., instructor in mathematics.
January 22, 1918: "The algebra of number pairs" by Grace Jefferson '18.
February 19: "The possible use of parallel axes in the plane of intersecting axes of coördinates" (continued)[1] by Professor Wiley.
March 5: "The different methods of defining the trigonometric functions and the identification of the functions so defined" (primarily for freshmen) by Professor Wiley.
March 19: "A problem in invariants" by Mildred Hunt '08.
April 16: "Introduction to the study of groups" by Professor Wiley.
May 14: Election of officers for 1918–19.

"During the past year our Club has felt increasingly the loss of strong members who have entered the national service."

UNDERGRADUATE MATHEMATICS CLUB, University of Illinois, Urbana, Ill.

The first mathematics club at the University of Illinois was organized during the year 1899–1900. Members of the faculty, graduate students and undergraduates constituted the membership. At the bi-weekly meetings for several years the topics discussed were suitable for undergraduates but gradually the club became an experiment station for trying out papers to be presented later before the American Mathematical Society. As a result in 1909 the Club was divided into two sections: (1) the graduate section with its tri-weekly meetings devoted mostly to the presentation of doctor's dissertations and original papers by faculty members; and (2) the undergraduate section which met monthly, for about an hour, "for the consideration of questions of general mathematical interest and the solution of problems." During several years the average attendance at meetings of this latter section was 30–35. While the management of the section was largely in the hands of graduate students a majority of the speakers were undergraduates. This arrangement did not seem to give entire satisfaction and still further bifurcation occurred in the autumn of 1917.

There are, then, now three organizations: I. Mathematics Club—Graduate Section, all members of the department staff and all graduate students being *eo ipso* members. The meetings are open to all interested and are devoted exclusively to reports on research work done in the department and to occasional reports on scientific meetings. II. The Mathematical Round Table, consisting of all graduate students in mathematics (and a few selected seniors) but not open to men with the Ph.D. degree. At the weekly meetings the average attendance is 12–15. Each member presents two papers a year on some semi-advanced topic not taught in the regular courses. Visitors are not encouraged. III. The Undergraduate Mathematics Club from which the department staff, as well as graduate students, are excluded. The members consist of juniors and seniors majoring in mathematics, and freshmen and sophomores of excellent stand-

[1] *Cf.* this MONTHLY, 1918, page 255.

ing in mathematics courses. All undergraduates are welcome as visitors. The average attendance is 8–12 from a membership of 36. Officers, 1917–18: President, Ruth Andrews '18; vice-president, Mary D. Craigmile '18; secretary, Harry W. Penhallow '19; program committee: the president, George Williams '18, and Dr. Aubrey J. Kempner (faculty adviser).

Apart from the solving of problems less difficult than those in the MONTHLY the programs of the Undergraduate Mathematics Club for last year were as follows.

January 23, 1918: "On paper folding" by Ruth Anderson '18.
February 6: "The value of high school mathematics" by Winifred White '18.
February 20: "Mathematical puzzles" by Fannie McMurray '19.
March 13: "On Rolle's Theorem" by George Williams '18.
April 3: "Constructions with the double ruler" by Mary Craigmile '18.
April 17: "The Theorem of Pythagoras" by Irene Doyle '19.
May 1: "The fundamental theorem of algebra" by C. T. G. Ching '18.
May 15: "On algebraic numbers" by Jesse E. Wilkins '18.

The programs of the Mathematics Club—Undergraduate Section—in 1915–16 were as follows.

November 7, 1915: "Who's who in mathematics in America" by Nathan C. Grimes, assistant in mathematics.
December 5: "Construction possibilities" by Katherine Lackey '16.
January 9, 1916: "Fourier Series and the harmonic analyzer" by Paul L. Bayley Gr.
February 13: "Nomography" by Joe Langueville Gr.
March 13: "Origin of calculus" by Erma Elliott Gr.
April 10: "Graphic solution of equations" by Albert E. Babbitt Gr.
May 9: "Mathematical models" by Mauritz Hedlund Gr.

THE MATHEMATICAL CLUB OF THE KANSAS STATE AGRICULTURAL COLLEGE, Manhattan, Kansas.

This club was organized in September, 1913 "to stimulate larger interest in mathematics on the part of students." Any student of collegiate rank was eligible for membership, which totalled 75 in 1916–17; the average attendance at meetings was about 20. Professor Benjamin L. Remick the head of the mathematics department acted as "officer in charge;" there were no student officers. "For various local reasons such as decrease in enrollment due to the war and the desire of several members of the department to engage in special mathematical work for themselves, it was decided not to carry on the usual club activities during 1917–18."

The programs for the first four years of the Club's existence were printed annually. They are reproduced below.

October 25, 1913: "Organizations and journals for the study of mathematics in America" by Professor Remick; "History of π" by Professor Alfred E. White.
November 8: "Euclidean constructions" by Professor William H. Andrews; "The history of logarithms" by Arthur Fehn, instructor.

November 22: "The teaching of secondary algebra" by Ina E. Holroyd, assistant; "The algebraic treatment of the evolute of a conic" by Professor Harrison E. Porter.

December 6: "History of a few proofs of the Pythagorem Theorem" by Charles H. Clevenger, instructor; "The place of limits in geometry" by Daisy D. Zeininger, instructor.

January 17, 1914: (*a*) "Note on problem 411, AMERICAN MATHEMATICAL MONTHLY,"[1] (*b*) "The perfect magic square for 1914" by Professor Remick; "The tendencies in modern mathematics" by Professor William T. Stratton.

January 31: "Concerning regular polyhedra" by Professor William H. Andrews; "The development of irrational numbers" by Professor White.

February 14: "The development of the decimal system" by Mr. Fehn; "Calculating machines" by Professor Porter.

February 28: "A mathematical notation applied to a few problems" by Mr. Clevenger; "The place of mathematics in the education of women and girls"[2] by Miss Holyroyd.

March 14: "Mathematics as a universal study" by Professor Stratton; "What is the laboratory method in the study of mathematics?" by Miss Zeininger.

April 11: "Non-Euclidean geometry" by Professor Remick; "Some fundamental ideas in mathematics" by Professor Andrews.

April 25: "The theory of duality" by Professor Porter; "The mathematical properties of maps" by Professor White.

May 9: "The history of trigonometry" by Mr. Fehn and Miss Zeininger; "Mathematics in English public schools" by Professor Stratton.

May 23: "The problem of failures" by Miss Holyroyd; "Coördinate systems" by Mr. Clevenger.

November 7: "Recent movements in mathematics" by Professor Remick; "Mathematical paradoxes" by Elliott Ranney '16; "History of the duplication of the cube" by Roy W. Haege '18.

November 21: "A problem in partial payments" by Professor Porter; "Graphical railroad time tables" by Gabe A. Sellers '17; "Plato as a mathematician" by Edith L. Alsop '16.

December 5: "Number bases among the primitive races" by Professor Stratton; "Mathematics at Kansas State Agricultural College from a student's standpoint" by William A. Lathrop '15; "History of algebraic symbolism" by Dilts S. McHugh '18.

January 16, 1915: "History of angular measurement" by Professor White; "History of the trisection of an angle" by Jefferson H. Flora '17; "Magic square for 1915" by Rufus S. Kirk '17.

January 30: "Short methods in the four fundamental operations" by Lee R.

[1] Vol. 20, p. 136: "*ABCD* is a rectangle of known sides. *BC* being produced indefinitely, it is required to draw a straight line from *A* cutting *CD* and *BC* in *X* and *Y*, respectively, so that the intercept *XY* may be equal to a given straight line. (Unsolved in *Educational Times*.)" Cf. *Proceedings of the Edinburgh Mathematical Society*, Vol. 28, pp. 152–178, 1909–10.

[2] Published in *School Science and Mathematics*, June, 1914.

Light '15; "Curves that trisect an angle" by Earl E. Swenson '17; "Origin of number symbols" by Caroline R. Packard '17.

February 13: "Number concept and generalization in algebra" by Mr. Fehn; "Mathematical games" by Zeno C. Rechel '18; "Elementary mathematics in evening schools" by Helen Mitchell '18.

February 27: "The mathematics of mineralogy" by Lyle M. Dean, assistant; "Life and works of Newton" by Louis R. Parkinson '16; "History of arithmetic in the United States" by Mae V. Hildebrand Sp.

March 13: "Correlation of mathematics and physics" by Eustace V. Floyd, assistant professor of physics; "History of the quadrature of the circle" by Robert F. Mirick '16; "Systems of quadratic equations" by James A. Hull '17.

April 10: "The logical and the psychological in geometry" by Miss Zeininger; "The controversy between Leibnitz and Newton" by Charles A. Willis '16; "Mystic properties of numbers" by Charles H. Zimmerman '16.

April 24: "Mathematical symmetry in nature"[1] by Miss Holyroyd; "Construction of logarithmic and trigonometric tables" by Donald D. Hughes '18; "Life and works of Descartes" by Russel H. Oliver.

May 1: "Thought versus rule in mathematics" by Joseph I. Kirkpatrick, assistant in veterinary medicine; "Euclid and his geometry" by Carl D. Hultgren '17; "Hypatia, last of the Greeks" by Sarah K. Kimport '18; "Geometry for engineers" by Leroy N. Miller '18.

May 15: "Historical solution of the quadratic equation" by Professor Andrews; "Relation of mathematics to statistics" by Lester Tubbs '17; "Geometry and construction work" by Basil A. Greene Sp.

November 6: "The place of mathematics in the study of heredity" by Edward N. Wentworth, professor of animal breeding.

November 20: "The function idea in and outside of mathematics" by Professor Remick; "The relation of mathematics to wireless telegraphy" by Harold M. McClelland '16; "Mathematics in verse" by Helen Mitchell '18.

December 4: "Generalizing some theorems in elementary geometry" by Professor Andrews; "The United States coast and geodetic survey" by Hubert A. Dawson '19; "The life and work of Poincaré" by Harry L. Robinson '18.

January 15, 1916: "The application of the prismatoid formula to the solids of elementary geometry" by Professor White; "History of arithmetic in the United States" by Gordon W. Hamilton '19; "The Pythagorean brotherhood" by Elizabeth A. Cotton '19.

January 29: Address by John O. Hamilton, professor of physics; "Zero and infinity" by Elliott Ranney '16.

February 12: "Means of measuring mathematical ability in students" by Professor Stratton; "Graphical solution of quadratic equations having complex

[1] In this connection, reference may be given to the somewhat unscientific work by S. Colman and C. A. Coan entitled *Nature's Harmonic Unity. A Treatise on its Relation to Proportional Form*, New York, Putnam, 1912.

roots" by Frank M. Sisson '18; "Proof and history of the fundamental theorem of algebra" by Otto B. Githens '17.

February 26: "The mathematics of investment" by Professor Porter; "The mathematics of chemistry" by Herbert H. King, assistant professor of chemistry.

March 4: "Mathematical symbolism and the economy of thought" by Miss Zeininger; "The solution of the biquadratic equation" by Charles A. Willis '16; "The problem of one cent" by Leroy N. Miller '18.

March 18: "The influence of French mathematics in America" by Miss Holyroyd; Problem discussion by Ivor O. Mall '18; "The solution of the cubic equation" by Andrew M. Harvey '18.

April 8: "History and development of Fermat's last theorem" by Mr. Fehn; "Some noteworthy series for the value of π and their derivation" by Lowell E. Baldwin '18; "Development of imaginary numbers" by Wilbur Lane '19.

April 22: Address by Siebelt L. Simmering, assistant professor of steam and gas engineering; "The cattle problem of Archimedes" by Charles A. Frankenhoff '19; "Two American mathematicians" by Earl V. Kesinger '17.

May 13: "The ancient and the modern treatment of proportion" by Lyle McF. Dean Gr. and assistant; "Mathematics and the science of war" by Jefferson H. Flora '17; Problem discussion by Joseph P. Ball '19.

November 14: "Recent developments in high school mathematics" by Miss Holyroyd.

December 5: "The mathematics of life insurance" by Professor Porter.

January 16, 1917: "The problem of three bodies—history and progress" by Professor Remick; "The construction of logarithmic tables" by Harry Dunham '18.

February 16: "Formulæ for the area of a triangle" by Miss Zeininger; "Some problems in gearing" by Myron R. Bowerman, assistant professor of mechanical drawing and machine design.

March 6: "Elementary geometry of the triangle" by Professor Stratton.

April 10: "Some fundamental ideas in mathematics" by Professor Andrews; "The application of mathematics to civil engineering" by Lowell E. Conrad, professor of civil engineering.

May 8: "Circles of the triangle" by Mr. Fehn; "The relation of mathematics to chemistry" by Herbert H. King, professor of chemistry.

June 5: "Construction of the polygon of seventeen sides" by Professor White; "Graphical solution of the quadratic with complex roots" by Donnelly J. Tarpey '19.

MATHEMATICS CLUB OF THE UNIVERSITY OF MONTANA, Missoula, Montana.

So far as the editor is aware this club was the first one to be founded in 1918. It resulted from a suggestion of its first president. Last semester anyone interested in mathematics could belong to the club but during the present year definite rules for members, as well as a statement of the purposes of the club, are

to be formulated. There were 22 members (3 of the faculty) last year and the average attendance was 18.

Officers 1917–18: President, Gretchen Van Cleve '19; secretary, Adele Maerdian '20; treasurer, Harry Rooney '21. The program committee consists of the president and secretary.

March 11, 1918: Organization of the club.

March 27: Address by Professor Nels J. Lennes, head of the department of mathematics.

April 10: "What other clubs are doing" by Doris Thetge '21; "Russian peasant method of multiplication and other methods" by Adele Maerdian '20.

April 24: "Method of constructing curves on roads and railroads" by Tom Swearingen '19; "Money value of an education" by Bessie Rutledge '20. Election of officers for 1918–19.

May 15: "Mathematics of war" by James Friauf '18.

May 29: "The slide rule" by Radcliffe Beckwith '21.

"At each one of our meetings we have had a 'feed' (a strictly war 'feed,' I assure you,—nothing unpatriotic about us out here in Montana). The young people seem to take interest and pride in getting up a good 'feed,' and they tax us an amount something like fifteen cents to a quarter, each time, to cover the expense. One Sunday we got four automobiles and drove about thirty miles into the country for an afternoon picnic."

THE MATHEMATICS CLUB OF NORTHWESTERN UNIVERSITY, Evanston, Ill. [*1918*, 132–134].[1]

March 7, 1918: "The history of the teaching of mathematics in the United States" by Mae Campbell '18; "Some mathematical fallacies" by Helen Maloney '18.

March 28: "Bomb throwing" by Frank D. Danielson '18.

April 18: "Gauss's method of quadratures" [subject of Master of Arts thesis] by Theodore Doll '17.

May 2: "Descriptive Geometry" by Milby R. Hammer.

May 18: Election of officers for 1918–19.

MATHEMATICAL CLUB OF ROCKFORD COLLEGE, Rockford, Ill. [*1918*, 187–188].

February 20, 1918: Debate, "Resolved that one year of mathematics should be required in college" by Dorothy Jamison '20, Virginia Schneider '20 (affirmative), and Lila Dole '20, Doris Volland '20 (negative).

March 6: Social meeting.

April 17: "Women in mathematics" by Estle Russell '18.

April 24: Special business meeting.

May 8: "Social meeting open to those freshmen who intend to go on with mathematics and who, therefore, will be able to join the club next year."

[1] This form of abbreviation will be used in the future to indicate earlier pages of this MONTHLY where other items concerning the club may be found.

THE NEWTONIAN SOCIETY OF THE STATE COLLEGE OF WASHINGTON, Pullman, Wash.

This society was founded as The Mathematical Society in November, 1911, but just five years later "after hearing claims made for more than a score of mathematicians, the society voted to name itself in honor of Newton."

"Its purpose is to afford an opportunity for students to form the habit of reading up on assigned topics which, for the most part, do not come up under regular courses. Broadly speaking, it is the aim of this society to lay the foundation for individual investigation and research. The students get some information and much inspiration from attendance upon these meetings. As a rule, one member of the faculty and one student take part in each meeting."

Any member of the faculty and any student interested in mathematics is eligible for membership. The total number of members last year was 14 and the average attendance at meetings was 10. "There are no fees, dues, or other items of expense, in connection with the society."

Officers 1917–18: President, Dorothy Neff '19; vice-president, Dorothea Sorenson '20; reporter (who looks after publicity in the college paper), Flossie Folsom '19; program committee, Professor Charles A. Isaacs, Florence Evans '20 and Rachel Shuman '18.

October 27, 1916: "The continuum" by Professor Isaacs.

November 10: "Life of Euclid" by Rachel Shuman '18; "Life of Archimedes" by Clarence L. Hix, instructor; "Life of Lagrange" by Professor Elmer C. Colpitts; "Life of Newton" by Marie Weldin '17; "Life of Euler" by Frank M. Bryant, instructor; "Life of Descartes" by Professor Isaacs.

November 24: "Mathematical Literature" by Elsie Worthen '20; "Mathematical books in the college library" by Rachel Shuman '18; "Mathematical periodical literature in the college library" by Professor Colpitts.

December 8: "Arithmetic and geometric progressions" by Corrine Barclay '19; "Convergent and divergent series" by Ina Craig '19; "Hypergeometric series" by Mr. Hix.

January 5, 1917: "Methods of Diophantus" by Edith McBride '19; "Arithmetica" by Mr. Bryant.

January 26: "The determinants in algebra" by Dorothy Neff '19; "The determinants in geometry" by Frank Hamelius '18; "Hessians and Jacobians" by Professor Isaacs.

March 2: "Permutations and combinations" by Blanche Lowary '20; "The theory of probability" by Flossie Folsom '19; "Insurance" by Professor Colpitts.

March 16: "Concurrent lines of a plane triangle" by Florence Evans '20; "Properties of a plane triangle" by Marie Weldin '17; "Circles connected with a plane triangle by Mr. Hix.

April 20: "Projectiles" by Rachel Shuman '18; "Gunnery" by Mr. Bryant.

October 18: "Points on a line" by Professor Isaacs; "Properties of a tetrastigm" by Ina Craig '19.

November 15: "Pythagorean astronomy" by Professor Colpitts; "Singular points on a curve" by Edith McBride '19.

December 13: "The ancient and modern abacus" by Mr. Hix.

January 24, 1918: "Symbolic logic" by Professor Isaacs; "Bertrand Russell" by Florence Evans '20.

February 25: "Some problems in modern geometry" by Mr. Bryant and Dorothy Neff '19.

March 21: "What, if any, mathematics should be required for graduates from secondary schools?" by Professor Isaacs; round table discussion by the whole society.

April 1: "Funny figures" by Amy Kelso '21; "Card tricks" by Mr. Hix.

April 11: "The sine law in plane geometry" by Dorothea Sorenson '20; "Addition and subtraction of logarithms" by Professor Colpitts.

May 13: "Arithmetical prodigies" by Flossie Folsom '19; "The nebular and other hypotheses" by Ina Craig '19.

May 16: "The history of the calculus" by Rachel Shuman '18; "Improper integrals" by Elsie Dallas '18.

"The number of programs for the past year was reduced so as not to interfere with the Red Cross work in which many students were engaged."

TOPICS FOR CLUB PROGRAMS.

14. The Cattle Problem of Archimedes.

During the last ten years of his life Gotthold Ephraim Lessing, the German critic and dramatist, occupied himself almost exclusively with the treasures of the library at Wolfenbüttel, Northern Germany, where he was librarian. The results of these researches were embodied in a series of volumes, *Beiträge zur Geschichte und Literatur*, the first being published at Braunschweig in 1773. In this volume first appeared the Greek epigram[1] (in verse form, 44 lines, from the

[1] Page 421 f. The text is followed by Lessing's commentary, a purported solution in Greek, by a scholiast, and a mathematical discussion by Christian Leiste (numerous misprints and errors in connection with the numbers). The same is to be found in the standard edition of Lessing's works: *Sämmtliche Schriften*, herausgegeben von K. Lachmann, besorgt durch F. Munker, Leipzig, Band 12, 1897, pp. 100–107, 110–115. (In this edition the misprints and errors of the 1773 edition have been corrected)—The Greek text of problem and scholium are also given (with Latin translation of the problem) by J. L. Heiberg, *Archimedes opera omnia*, iterum edidit, Vol. 2, Lipsiæ, MDCCCCXIII, pp. 528–534. (See also commentary in J. L. Heiberg, *Quæstiones Archimedeæ*, Hauniae, MDCCLXXIX, pp. 26f., 66f.)—For German translations see: (1) G. F. Nesselmann, *Die Algebra der Griechen*, Berlin, 1842, pp. 481–491; (2) B. Krumbiegel, "Das problema bovinum des Archimedes," *Zeitschrift für Mathematik und Physik*, hist. literar. Abt., 1880, Vol. 25, pp. 121–136. (This comprehensive discussion of earlier work contains German translations of both epigram and scholium. Krumbiegel's paper is followed, pages 153–171, by A. Amthor's mathematical discussion. The two present a masterly presentation of the facts and are fundamental for every student of the problem.)—French translations by Terquem and A. S. C. Vincent in *Bulletin de bibliographie, d'histoire et de biographie mathématiques*, tome 1, 1855, pp. 113–124, 130, 165–173; tome 2, 1856, pp. 39–42.—Italian translation by G. Loria in *Le scienze esatte nell' antica Grecia*, 2a ed., Milano, 1914, pp. 932–939 (Il "problema dei buoi" di Archimede).

The cattle problem was also carefully studied by:

(*a*) J. Struve and K. L. Struve, Vater und Sohn, *Altes griechisches Epigramm mathematischen*

original[1] discovered by Lessing in the Wolfenbüttel library) headed[2]: "A problem which Archimedes found among (some) epigrams and sent, to be solved by those in Alexandria who occupy themselves with such matters, in his letter to Eratosthenes of Cyrene."

In abbreviated and partly symbolic form the problem is as follows: Compute, O friend, the host of the oxen of the Sun, giving thy mind thereto, if thou hast a share of wisdom, compute the number which once grazed upon the plains of the Sicilian isle Thrinacia,[3] and which were divided according to color into four herds, one milk white, one black, one yellow and one dappled. The number of bulls formed the majority of the animals in each herd and the relations between them were as follows:

1 . . . White bulls (W) $= (1/2 + 1/3)$ black bulls (X) + yellow bulls (Y),
2 . . . Black bulls (X) $= (1/4 + 1/5)$ dappled bulls (Z) + yellow bulls (Y),
3 . . . Dappled bulls (Z) $= (1/6 + 1/7)$ white bulls (W) + yellow bulls (Y),

As to the cows:

4 . . . White cows (w) $= (1/3 + 1/4)$ black herd ($X + x$),
5 . . . Black cows (x) $= (1/4 + 1/5)$ dappled herd ($Z + z$),
6 . . . Dappled cows (z) $= (1/5 + 1/6)$ yellow herd ($Y + y$),
7 . . . Yellow cows (y) $= (1/6 + 1/7)$ white herd ($W + w$).

Inhalts von Lessing erst einmal zum Drucke befördert, jetzt neu abgedruckt und mathematisch und kritisch behandelt, Altona, 1821 (47 pp.).

(*b*) J. G. Hermann (the German classical scholar and philologist), *De archimedis problemate bovino*, Leipzig, 1828 (12 pp.). (Also in *Godofredi Hermanni Opuscula*, Vol. 4, Lipsiæ, 1831, pp. iii–v, 228–238; contains some notes not in original pamphlet.)

(*c*) P. Tannery: (1) "Sur le problème des boeufs d'Archimède," *Bulletin des sciences mathématiques et astronomiques*, 1881, tome 5, part 1, pp. 25–30 (also in P. Tannery, *Mémoires scientifiques*, tome 1, Toulouse, 1912, pp. 118–123); (2) "L'arithmétique des grecs dans Pappus" (1880), *Mémoires scientifiques*, tome 1, 1912, pp. 103–105.

(*d*) F. O. Hultsch im Pauly-Wissowa's *Real Encyclopädie der Classischen Altertumswissenschaft*, Band 2, Stuttgart, 1896, cols. 531–535, 1110.

Students may find T. L. Heath's somewhat brief discussions more accessible: (1) *The Works of Archimedes*, Cambridge, 1897, pp. xxxiv–xxxv, 319–326. (Also in *Archimedes' Werke mit modernen Bezeichnungen herausgegeben und mit einer Einleitung versehen* von Sir Thomas L. Heath. Deutsch von F. Kleim. Berlin, 1914, pp. 471–477). (2) *Diophantus of Alexandria, A study in the History of Greek Algebra*, 2d edition, Cambridge, 1910, pp. 11, 12, 121–124, 279.

[1] There is another copy of the epigram in the Bibliothèque nationale, cod. Paris Gr. 2448 saec. XIV, fol 57. Heiberg indicates all the small differences of the codices.

[2] In this connection I have chosen the translation which Krumbiegel and Heath regard as literal, though somewhat unsatisfactory in meaning.

[3] The problem seems to hark back to the twelfth book of Homer's *Odyssey* where the following lines occur: "Next, you will reach the island of Thrinacia where in great numbers feed kine and the sturdy flocks of the Sun" (G. H. Palmer's translation, Boston, 1894, pp. 188–189); or in Pope's translation

"Thence to Trincaria's shore you bend your way,
Where graze thy herd, illustrious source of day!"

As to the name τρινακρία for Sicily, and the θρινακίη of Homer and of the epigram, see *Encyclopaedia Britannica*, article "Sicily," and Homer's *Odyssey* edited by W. W. Merry and J. Riddell, 2d ed., Vol. 1, Oxford, 1886, p. 516.

If thou canst give, O friend, the number of bulls and cows in each herd thou art not unknowing nor unskilled in numbers, but still not yet to be counted among the wise.

Consider, however, the following additional relations between the bulls of the Sun:

8 . . . White bulls (W) + black bulls (X) = a square,
9 . . . Dappled bulls (Z) + yellow bulls (Y) = a triangular number.

When thou hast then computed the totals of the herds, O friend, go forth as conqueror, and rest assured that thou art proved most skilled in the science of numbers.

The first seven equations, in eight unknowns, lead to the solution:

$$W = 10366482\,n,\ X = 7460514\,n,\ Y = 4149387\,n,\ Z = 7358060\,n,$$
$$w = 7206360\,n,\ x = 4893246\,n,\ y = 5439213\,n,\ z = 3515820\,n,$$

where n is any integer. The solution given without explanation by the scholiast corresponds to $n = 80$, and the total number of cattle of the Sun 4,031,125,560. These numbers do not, as he affirms, satisfy conditions 8 and 9.

It is generally accepted that condition 8 is equivalent to $W + X =$ a square *number*; but an ambiquity in the language of the epigram in this connection makes possible the strained interpretation that $W + X =$ a square *figure* or rectangle (i. e. the product of two factors), since a bull is longer than it is broad.

With this latter interpretation of condition 8 the problem was completely solved by J. F. Wurm in 1830,[1] and the total number of cattle of the Sun was then found to be 5,916,837,175,686. If it were desirable to distribute these cattle uniformly over all the dry land of the earth, one animal would come about every 25 square feet.

Amthor attacked the problem on the basis of the more generally accepted reading and this called for the solution of the equation

$$t^2 - 4729494\,u^2 = 1^2$$

After elaborate and arduous work, especially with continued fractions, he arrived at the conclusion that $W = 1598\ \langle 206541 \rangle$, where $\langle 206541 \rangle$ represents the fact that there are 206541 more digits to follow, and that, with the same notation the whole number of cattle = $7766\ \langle 206541 \rangle$. "It is easy to show that a sphere having the diameter of the milky way, across which light takes ten thousand years to travel, could contain only a part of this great number of animals even if the size of each is that of the smallest bacterium."

There has been much debate as to whether Archimedes really propounded the

[1] Jahn's *Jahrbücher für Philologie und Pädagogik*, Vol. 14, 1830, p. 194 f. Review of Hermann's pamphlet. Amthor gives the solution of Wurm's problem.

[2] This equation has been referred to as a Pellian equation, but there is not the slightest ground for so designating any equation of this form. See *Encyclopédie des sciences mathématiques*, tone 1, Vol. 3, fasc. 1, 1906, p. 27.

cattle problem. In 1880 Krumbiegel gave a pretty complete account of arguments for and against this theory. As a result he concludes: The exact form of the problem is probably later than Archimedes, but as to the problem itself, not only is it very possible, but very probable that it really originated with the celebrated geometer of Syracuse. Such too is the opinion of Tannery, and of Heiberg[1] who is the greatest authority on the texts of Archimedes. Hultsch discusses "a most attractive suggestion" (Heath) that the unmistakable vein of satire in the opening words of the epigram, in the transition from the first to the second parts, and in the last lines, was a shaft directed towards Apollonius.

This problem, originating before the beginning of the Christian era is remarkable in the history of Greek algebra since it is a problem in indeterminate analysis of the second degree, a problem the solution of which is more complicated than that of any in extant works of Diophantus.

K. B. Mollweide (who was born in Wolfenbüttel) is quoted as authority for the statement that his friend Gauss had solved the Cattle Problem completely.[2] If Gauss really obtained the solution he is the only person known to have done so. For, although it has been held very unsafe to claim that Archimedes had at his disposal no method sufficiently powerful to cope with such a question, no one contends (nor was it necessary for giving to Apollonius his quietus in accordance with Hultsch's suggestion) that he actually carried through all the necessary computations.

As final reference to the literature of our topic I note three papers published in America, two by A. H. Bell,[3] and one by Mansfield Merriman[4] to which the student who is unable to read the admirable and comprehensive German memoirs of Krumbiegel and Amthor will naturally turn. Bell's papers set forth the results of nearly four years of computation by himself and two others who constituted the "Hillsboro Mathematical Club" of Hillsboro, Ill. They computed thirty or thirty-one of the left-hand figures and twelve of the right-hand figures for each of the eight unknowns, as well as for the total number of the Cattle of the Sun. Amthor seems to have contented himself with computing the first four of the left-hand figures of (*a*) one of the unknowns, (*b*) the total (although similar values for other unknowns follow readily from his work). The corresponding printed results of Bell and Amthor do not agree.

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[1] *Quæstiones Archimedeæ*, l. c., and *Philologus*, Göttingen, Band 43, 1884, p. 486.

With these authorities Heath too seems to range himself. It is interesting to compare his discussion of the Cattle Problem in the first edition of his *Diophantos of Alexandria*, Cambridge, 1885, pp. 7, 142–147, with that given in the second.

[2] Hermann, *l. c.*, p. 230 of *Opuscula*.

[3] (1) "On the celebrated 'Cattle Problem' of Archimedes," *The Mathematical Magazine*, Washington, January, 1895, Vol. 2, pp. 163–164; (2) "The 'Cattle Problem' by Archimedes 251 B. C.," AMERICAN MATHEMATICAL MONTHLY, May, 1895, Vol. 2, pp. 140–141.

[4] M. Merriman, "The Cattle Problem of Archimedes," *Popular Science Monthly*, 1905, Vol. 67, pp. 660–665.